The Research Foundation for Accelerated Math™
Accelerated Math has "strong evidence of effectiveness" for prevention and intervention at all grade levels according to the National Dropout Prevention Center/Network.

Accelerated Math is highly rated for progress monitoring mastery measurement by the National Center on Response to Intervention.

Accelerated Math is highly rated for progress monitoring mastery measurement by the National Center on Intensive Intervention.

Accelerated Math meets the technical standards of a progress monitoring tool according to the National Center on Student Progress Monitoring.
Introduction
Extensive research has determined which instructional practices have the greatest impact on student achievement. Providing students time for appropriately challenging math practice, ensuring students receive feedback on this practice, setting goals with students, and frequently monitoring students’ progress toward set goals are all components of effective instruction and supported by a well-confirmed knowledge base (e.g., Christenson & Ysseldyke, 1989; Ysseldyke & Christenson, 1987; see Appendix A, p. 18).

Practice is a non-negotiable part of the learning process; this is especially true in mathematics. Time spent intensively practicing a skill—not initial ability—is the single most important shared characteristic of mathematicians as well as world-class chess players and musicians (Coyle, 2009; Ericsson, Charness, Feltovich, & Hoffman, 2006). Per cognitive scientist Dan Willingham, “It is virtually impossible to become proficient at a mental task without extended practice” (2009, p. 81).

In addition to practice, progress monitoring with formative assessment is necessary to continuously check the effectiveness of classroom instruction, identify students struggling with math skills, and measure students’ growth toward personal goals. “Only by keeping a very close eye on emerging learning through formative assessment can teachers be prospective, determining what is within the students’ reach, and providing them experiences to support and extend learning” (Heritage, 2010, p. 8).

The purpose of formative assessment is to provide feedback to students that will help close the gap between current academic performance and goal performance. To do this, “teachers need to have in mind a continuum of how learning develops in any particular knowledge domain so that they are able to locate students’ current learning status and decide on pedagogical action to move students’ learning forward” (Heritage, 2008, p. 2). Clearly articulated learning progressions can support instructional planning and formative assessment.

Although the need for practice and progress monitoring in building various skills is clear, these instructional practices are not always implemented in the classroom. The National Mathematics Advisory Panel (National Math Panel [NMP], 2008b) report states that “few curricula in the United States provide sufficient practice” (p. 26). Additionally, research on classroom-level monitoring and assessment reveals that “many teachers do not: assign homework frequently or regularly, record completion assignments, monitor seat work and check on students’ progress, or conduct the kind of questioning that helps to monitor learning” (Khairou, 2014, p. 35). As for learning progressions, Heritage (2008) notes that “teachers need to understand the pathways along which students are expected to progress...despite a plethora of standards and curricula, many teachers are unclear about how learning progresses in specific domains” (p. 2).

Effective, personalized practice requires continuous daily—even hourly—instruction, practice, and assessment within a formative assessment process. Accelerated Math facilitates student math practice and equips teachers with data to monitor progress, adjust instruction and curriculum, and provide feedback.

Why math practice is essential
Mathematical proficiency is of critical importance, both to succeed academically and ultimately in life beyond schooling. The National Research Council (2001) uses this term to describe “five interrelated strands of knowledge, skills, abilities, and beliefs that allow for mathematics manipulation and achievement across all mathematical domains (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition)” (Steedly, Dragoo, Arefeh, & Luke, 2008, p. 10). For students to learn successfully, these interconnected strands must work together. “For example, as a student gains conceptual understanding, computational procedures are remembered better and used more flexibly to solve problems. In turn, as a procedure becomes more automatic, the student is enabled to think about other aspects of a problem and to tackle new kinds of problems, which leads to understanding” (National Research Council, 2002, p. 17).
Students are capable of learning mathematics, but it does not just happen—it takes time, effort, and practice (Willingham, 2009–10). Practice is an essential part of learning and is often what makes the difference between successfully learning a skill or not: “It is intuitively obvious that practice is necessary for learning knowledge of any type. It’s not surprising then that research indicated that practice significantly enhances learning….The effect of practice on learning can be substantial” (Marzano, Gaddy, & Dean, 2000, p. 66).

Brain research explains why practice is universally important for learning. Practice helps build neurological connections, evidenced by increased white matter in the brain, which is the result of a process called myelination (see figure 1). These connections help people solve higher order problems faster and more efficiently, using less area of the brain and with less overall brain activity. Once a skill is taught, repeated practice alters the neurons in a part of the brain specific to that skill, so a skill can be performed automatically, seemingly without thought (Coyle, 2009; Hill & Schneider, 2006). “Practice allows students to achieve automaticity of basic skills—the fast, accurate, and effortless processing of content information—which frees up working memory for more complex aspects of problem solving” (NMP, 2008b, p. 30; Sousa, 2006).

**Figure 1. Practice thickens myelin sheath**

Research shows practice increases neurological connections, specifically the thickness of insulating myelin sheath, which vastly improves overall brain efficiency and higher-order thinking ability.

Researchers say we forget much of what we learn, and that the forgetting is rapid, though continued practice can protect against forgetting (Willingham, 2009). In one study, researchers found that a student who achieved a C in his first algebra course but went on to take several more math courses remembered his algebra, whereas a student who got an A in algebra but did not continue in math courses forgot what he learned (Bahrick & Hall, 1991). Taking additional math courses ensures students continue to think about and practice what they have learned.

Research also shows that in order to develop competence in skills, students need to adapt or “shape” them as they are learning. Appropriately challenging personalized practice with feedback is the primary means of doing this. The timing of feedback is of key importance, as students should have the opportunity to grapple with a problem and self-correct, finding any missteps through additional practice and error analysis. According to Marzano et al. (2000):

> During this shaping phase, learners modify the way they use the skill, become aware of potential problem areas as well as variations in how the skill can be used, and learn to use the skill in different situations. The importance of this shaping phase cannot be overstated, yet this crucial stage of learning is often not given the necessary time and attention. Skipping or shortchanging this stage of learning can result in students’ internalizing errors that are difficult to correct. It can also mean that students will not gain the conceptual understanding that is essential to truly mastering a skill or process. (p. 67)

Academic learning time (ALT)—the amount of time students spend on actual learning activities—has long been identified as a critical contributor to academic growth (Batsche, 2007; Berliner, 1990; Gettinger & Stoiber, 1999; Karweit, 1982). An important, but often underemphasized aspect of ALT is time for practice of learned skills—which is as important as explicit instruction (Szadokierski & Burns, 2008). Since ALT is the time when most learning actually takes place, increasing ALT is a powerful tool.

Four components distinguish ALT from simple measures of classroom time or “time on task”:

- Students are engaged with material
- Material is at the proper level of challenge
- Students experience high rates of success
- Both students and teacher receive regular feedback about performance

The challenge is that to increase ALT it is necessary to directly manage it, which is exceedingly difficult, especially in diverse classrooms where learners are working at many different levels at the same time.

The role of practice within new college- and career-readiness standards

The role of practice in learning mathematics is especially crucial in light of new college- and career-readiness standards, which require “students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks” (Hess, Carlock, Jones, & Walkup, 2009, p. 1). The rigor required of the new standards demands pedagogical shifts (Engage NY, 2012) that relate to researcher Norman Webb’s widely accepted measure of cognitive rigor, the four depths of knowledge (DOK) levels: Level 1: Recall & Reproduction, Level 2: Basic Skills & Concepts, Level 3: Strategic Thinking & Reasoning, and Level 4: Extended Thinking. The four levels each play an important role in students’ learning and do not represent a linear progression.

Integrating deeper levels of knowledge is important, so that students are able to strategically apply concepts learned to other problems. However, it is a common misconception that students should be spending all or most of their time on higher-order thinking skills and less time on learning and practicing foundational skills. Particularly in mathematics, there should be strong emphasis on foundational skills so that students can successfully move on to application of these skills.

Descriptive analyses of the Common Core State Standards prepared for the Smarter Balanced Assessment Consortium show the following breakdown of DOK levels assigned to the new mathematics standards (each standard can cover a range of DOK levels): 89% were assigned DOK Level 1, 79% were assigned DOK Level 2, 21% were assigned DOK Level 3, and less than 1% were assigned DOK Level 4 (Sato, Lagunoff, & Worth, 2011). And practice guru and author Doug Lemov argues that although many educators perceive “drilling” students with foundational skills is the enemy of higher-order thinking, cognitive psychology research shows this is not true (Lemov, Woolway, & Yezzi, 2012). Establishing a strong foundation in basic skills paves the way for future understanding, application, and analysis of concepts learned. According to Willingham (2009), basic processes that initially are demanding of working memory become automatic with practice, and by making these low-level processes automatic, room is made for more high-level concerns.

Establishing a strong foundation in basic skills paves the way for future understanding, application, and analysis of concepts learned.

Accelerated Math™ software provides solutions for educators’ biggest challenges

Accelerated Math provides efficient progress monitoring and management of students’ personalized daily math practice for grades K–12 within a formative assessment process. Accelerated Math can be used successfully as the essential student practice component of existing classroom mathematics curricula (Ysseldyke & Betts, 2010; Ysseldyke & Bolt, 2007), supplying libraries of math problems from kindergarten math to calculus, algebra, geometry, and probability and statistics, as well as libraries aligned to various state standards, national guidelines, and textbook series. The program assists teachers with

- Creating personalized assignments and tests at an appropriate level for each student
- Scoring student practice and assessments automatically, and recording results in a grade book
- Providing informative feedback to help educators differentiate instruction and monitor progress
- Encouraging mathematical discourse and collaboration
- Motivating students with immediate feedback after assignments or tests are completed
• Promoting practices that ensure fidelity of implementation, which include recommendations for optimal growth and appropriate goal-setting practices (see Appendix B, p. 19)

The Accelerated Math libraries were first published in 1998, with a scope and sequence based on commonalities between the 1989 National Council of Teachers of Mathematics (NCTM) standards and leading publisher textbooks, National Assessment of Educational Progress (NAEP) editions, and math editor teaching experience from the 1990s. Then in 2006, the Accelerated Math libraries went through a thorough review and update to create what we termed the Second-Edition Libraries. To ensure the content in the second edition met the highest standards of the day, Renaissance Learning developed a new scope and sequence for early numeracy, grades 1 through 8, algebra 1, and geometry that included core objectives, learning progressions, and prerequisite skills.

Several resources informed the new scope and sequence: extensive research by Renaissance on the NCTM Curriculum Focal Points (2006), the final report of the National Mathematics Advisory Panel (2008a, 2008b), and state and international standards; convening of a reviewer panel of mathematicians and researchers including Dr. Sybilla Beckmann (University of Georgia), Dr. Richard Bisk (Worcester State College), Dr. Tom Hogan (University of Scranton), Dr. R. James Milgram (Stanford University), Dr. Sharif M. Shakhrami (private consultant/researcher), and Dr. Amanda VanDerHeyden (private consultant/researcher); and consultation with the US Department of Education’s Northwest Regional Educational Laboratory and mathematics teachers in several states.

Then in 2013, a significant edition to Accelerated Math came in the form of custom-built libraries built specifically aligned to college and career-readiness standards. Renaissance’s content development team created the brand-new content following constructs in the K–8 and High School Publishers’ Criteria for the Common Core State Standards for Mathematics, including identifying and analyzing skills, developing a coherent progression (validated with data on student performance), ensuring content adhered to CCSS expectations, and engaging with mathematics experts for input. Dr. VanDerHeyden (grades K–2), Dr. Karen Hess, private consultant/researcher (grades 3–5), and Dr. Milgram (grades 6–high school) provided expert review of the libraries prior to publication.

Since that time, further refinements to Accelerated Math help teachers balance math practice both for students working on grade level standards and for those striving to catch up on foundational skills. Students are grouped according to STAR Math data, and the Accelerated Math then schedules standards coverage and dynamic practice for the entire school year. Teachers monitor progress and manage assignments, which helps to inform tactics for differentiated instruction and move each student toward deep standards mastery.

Content aligned with key math recommendations

Accelerated Math facilitates formative assessment defined by McManus (2008) as “a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of intended instructional outcomes” (p. 3). Likewise, the software helps educators meet recommendations in the National Math Panel’s final report (2008b) by

- Presenting “a focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics” (pp. xvi, 20–22)
- Supplying “sufficient and appropriate practice” that fosters “computational proficiency with whole number operations….fluency with the standard algorithms….and a solid understanding of core concepts” (pp. xix, 26–29)
- Focusing on effort rather than ability, which “increases [student] engagement in mathematics learning [and] improves mathematics outcomes” (pp. xx, 31–32)
- Promoting “regular use of formative assessment,” which “improves…students’ learning” (pp. xxiii, 46–48)
- Providing “tools that inform teachers about specific ways of using formative assessment information to provide differentiated instruction” (pp. 46–48)

Accelerated Math meets five criteria identified by the FAST SCASS for effective formative assessment (McManus, 2008):

1. Learning progressions
2. Learning goals and criteria for success
3. Descriptive feedback
4. Self- and peer-assessment
5. Collaboration

Accelerated Math also meets five criteria identified by the Formative Assessment for Students and Teachers (FAST) State Collaborative on Assessment and Student Standards (SCASS) for effective formative assessment: (1) learning progressions, (2) learning goals and criteria for success, (3) descriptive feedback, (4) self- and peer-assessment, and (5) collaboration (McManus, 2008).
Even though personalized practice and progress monitoring are key to student success in mathematics, classrooms may lack these routines due to three common barriers (see table 1). Accelerated Math can help educators meet these challenges.

Table 1. Accelerated Math provides solutions for common barriers to practice and progress monitoring

<table>
<thead>
<tr>
<th>Challenge for educators</th>
<th>How Accelerated Math helps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effectively and efficiently personalizing student practice</td>
<td>Accelerated Math helps teachers analyze individual skills deficiencies and fill gaps in learning progressions as well as increase student practice of specific standards-linked skills. The National Math Panel describes Accelerated Math as a “mathematics program with assessment of skill level, tailoring of the instruction to match skill level, individual pacing and goal setting, ample practice, and immediate feedback to student and teacher on performance” (2008a, p. 160).</td>
</tr>
<tr>
<td>Consistently monitoring student progress</td>
<td>As a technology-enhanced, continuous progress-monitoring system (Ysseldyke &amp; Burns, 2009; Ysseldyke &amp; McLeod, 2007), Accelerated Math can be used for practice progress monitoring—the measuring of performance by underlying tasks (such as math problems) that contribute to growth in math skills or benchmarks.</td>
</tr>
<tr>
<td>Bridging assessment and instruction</td>
<td>The Core Progress for math learning progression (see pp. 7–8, also Renaissance Learning, 2013) reveals intermediate steps and prerequisite skills needed to reach the level of expertise required by college- and career-readiness standards. Core Progress skills and understandings range from early numeracy to competencies required in college and the workplace.</td>
</tr>
</tbody>
</table>

Facilitates efficient and effective personalized student practice

Accelerated Math helps educators facilitate academic learning time by supporting key aspects of effective student practice (see table 2).

Table 2. Accelerated Math accomplishes key components of effective practice

<table>
<thead>
<tr>
<th>Characteristics of effective practice</th>
<th>What Accelerated Math does</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully engaging with the material</td>
<td>The Accelerated Math classroom is structured so that students are always using time productively, by working on assignments or tests, submitting work to be scored, or conferring with the teacher for additional instruction and intervention or with other students for peer-learning opportunities. Student routines and expectations are clearly communicated in an Accelerated Math classroom, so students always know where efforts should be focused. If guidance is needed, the teacher (and classroom signage) is available to provide direction.</td>
</tr>
<tr>
<td>Working at an optimal level of challenge</td>
<td>STAR Math assessments coupled with ongoing measurement of results from the Accelerated Math program ensure assignments are tailored to each student’s individualized level, which is continuously recalculated as the student progresses.</td>
</tr>
<tr>
<td>Students experiencing a high rate of success</td>
<td>With Accelerated Math, teachers help students become highly successful math learners by ensuring they maintain an average percent correct of 75% or above on assignments and 85% or above on tests.</td>
</tr>
<tr>
<td>Receiving immediate informative feedback</td>
<td>Instant feedback is provided to both students and teachers in Accelerated Math, making it possible for students to know exactly which subskills need more practice and for teachers to monitor progress and adjust assignments according to student needs. By design, feedback is shared after completed assignments and tests, rather than after each item, to give students an opportunity to self-correct, either through repeated practice or error analysis.</td>
</tr>
</tbody>
</table>

Evidence of the positive effect of time spent by students engaged in learning activities (i.e., academic learning time) on student growth is revealed in an analysis of the Accelerated Math database. Figure 2 (next page) shows that students who master more math subskills per week have higher student growth percentiles (SGPs), a measure that compares a student’s growth to that of academic peers nationwide (i.e., students in the same grade with similar pretest scores).
**Encourages student collaboration**

Talking about math encourages students to take ownership of their practice and reflect on new math concepts. Encouraging mathematical discourse in the classroom allows students to make sense of and critique their peers’ ideas, creating deep, connected math knowledge that will prepare them for the rigors of college and the workplace.

Students’ Accelerated Math assignments are an ideal jumping off point for students to talk with their classmates about the math skills they are learning. The work they do on paper while solving unique problem sets creates a record of their reasoning they can reference while speaking with their peers. Students can share the steps they took to solve a particular problem and share points of confusion or road blocks they encountered while completing their work.
High-quality progress monitoring and mastery measurement with Accelerated Math™

Accelerated Math can be thought of as an assessment for daily practice monitoring, meaning it is designed to provide feedback regarding both student completion of important tasks known to improve achievement outcomes (such as math problem solving) and student comprehension of direct instruction. Assessments at this level provide the majority of the information necessary to inform instruction and personalize student practice that will improve performance. Integrated assessment technology is a practical way to administer daily assessments and benefit from the wealth of data they provide (Renaissance Learning, 2011).

Given the strong connection between teacher monitoring of student progress and subsequent academic performance for students, it is critical teachers receive thorough training in progress monitoring. However, a meta-analysis of classroom-level monitoring and assessment research reveals that this is not always the case (Cotton, 1988):

- Many teachers do not assign homework frequently or regularly, record completion assignments, monitor seatwork and check students’ progress, or conduct questioning that helps to monitor learning.
- Teachers do not receive adequate preservice training in conducting formal or informal assessments.
- Administrative support for and in-service training in the skills associated with assessment and monitoring are extremely insufficient.
- Many teachers are aware their monitoring skills are inadequate and desire training to expand their capabilities; others are unaware of the importance of close monitoring of student progress and of their own need for skill development.

Research on teachers’ decision-making processes suggests a lack of progress monitoring. According to the research, “a great many teachers are reluctant to make changes in the instructional strategy or pacing of lessons once these are planned, even when instruction and learning are progressing poorly” (Cotton, 1988, p. 6).

Accelerated Math provides continued, differentiated student practice within a range of difficulty that is neither too difficult, and thus frustrating, nor so easy that no new knowledge and skills are learned. In order to customize student practice and ensure students are working at appropriate levels of challenge, the teacher determines each student’s initial placement in Accelerated Math using results from a reliable and valid, standardized math assessment, such as the STAR Math assessment, and/or an Accelerated Math Diagnostic Test. Based on the results, students begin personalized practice/test cycles that progress through mastery and review of each needed subskill. In mastery measurement systems, students work through a hierarchical sequence of skills and demonstrate mastery of each before proceeding to the next skill (www.rti4success.org).

With Accelerated Math, student math practice and teacher monitoring take place each day. This continuous daily stream of data shows how each student is progressing and provides feedback long before progress can be measured by other methods, such as weekly or monthly progress monitoring. The program tracks the status of each student’s practice assignments and tests, including the subskills a student or group has been assigned, are ready to test on, or may be struggling with. Also tracked are students’ percent correct on assignments and tests, number of subskills mastered, and academic growth and achievement. These insights provide a quick but holistic view of students’ overall math progress, which helps teachers evaluate instruction, identify student needs, and intervene quickly and effectively to give all students help exactly when needed. Immediate feedback is much more useful to inform and adapt instruction and student practice than waiting for mid-year or end-of-year scores when it is too late to make changes that could impact a student’s growth.

Bridging assessment and instruction with Accelerated Math™

Renaissance’s trusted Accelerated Math software has impacted student mathematics learning and provided a way for teachers to measure mastery of math skills for more than 15 years. As described by the National Math Panel, Accelerated Math is a “mathematics program with assessment of skill level, tailoring of the instruction to match skill level, individual pacing and goal setting, ample practice, and immediate feedback to student and teacher on performance” (2008a, p. 160).

Learning progressions

New standards for teaching and learning abound in education today; however, as Duschl, Schweingruber, and Shouse (2007) note, “many standards and curricula contain too many disconnected topics that are given equal priority. The way many
standards and curricula are conceived limits their utility for planning instruction and assessing learning. Too little attention is given to how students’ understanding of a topic can be supported from grade to grade” (p. 231). Explicit learning progressions can provide clarity for teachers by describing a pathway of learning for each student that will assist with instruction.

Learning progressions describe how learning typically advances in a subject area. “Empirically based learning progressions can visually and verbally articulate a hypothesis, or an anticipated path, of how student learning will typically move toward increased understanding over time with good instruction” (Hess, Kurizaki, & Holt, 2009). This pathway anchors both instruction and assessment because “when teachers understand the continuum of learning in a domain and have information about current status relative to learning goals (rather than to the activity they have designed to help students meet the goal), they are better able to make decisions about what the next steps in learning should be” (Heritage, 2008, p. 2).

Recent enhancements to STAR Math and Accelerated Math seamlessly integrate assessment with instruction and practice. A student takes a STAR Math test, and the resulting score helps teachers determine the level of understanding of particular skills and the type of practice needed. Accelerated Math then generates personalized practice assignments based on the skills the student is ready to learn next or to review. The order of the subskills assigned to each student is based on an interconnected web of prerequisite skills in the Core Progress for Math learning progression.

Core Progress was developed to provide a research-based framework for Accelerated Math (Renaissance Learning, 2013). Once built, Core Progress skills were field tested through the STAR Math assessment, with remarkable results. As illustrated in figure 3, the order of skills in Core Progress was highly correlated with the difficulty level of STAR Math items. With this strong correlation, the natural next step was to statistically link Core Progress to STAR Math.

**Figure 3. Core Progress skill difficulty highly correlates with STAR Math items**

As a result of this linking, a student’s STAR Math score provides insight into both achievement level and the skills and understandings the student is ready to develop next. Core Progress is now an integral component of Accelerated Math and STAR Math, helping to seamlessly bridge assessment, instruction, and practice.

**Can be used with any curriculum**

No matter the curriculum used in a classroom, Accelerated Math is an excellent means for providing and monitoring student math practice. Teachers deliver instruction using a given textbook, and can then have students use Accelerated Math to practice what they have learned.

Content aligned to college- and career-readiness standards
To help students achieve deep standards mastery, Accelerated Math’s K–12 content libraries are built to meet the rigors of new college and career expectations and are differentiated by depth. The program focuses on providing a solid base in foundational skills as well as practice with higher-order thinking skills.

Using the Core Progress learning progression as a guide, Accelerated Math automatically schedules assignments covering appropriate standards and exposing students to varying levels of practice. Core Progress provides information about intermediate steps and prerequisite skills that are necessary to reach the level of expertise identified through the standards, beginning with early numeracy and progressing through the type of knowledge required in college and the workplace.

Use with Response to Intervention (RTI) and differentiated instruction
A key guiding principle of Response to Intervention is differentiated instruction—at all tiers—with personalized goal setting that allows teachers to accurately monitor students’ growth in a timely manner and make changes to instruction as necessary. Differentiated instruction in RTI should not be limited to students formally designated to receive interventions—it should apply within the core (Tier 1) classroom as well. It is true that differentiated instruction is difficult—because it inherently implies setting, and monitoring, individual goals.

After placement in Accelerated Math based on a student’s score on an Accelerated Math Diagnostic Test, a STAR Math assessment, or both, the software automatically creates assignments geared directly to that student’s individual levels and appropriate subskills. This process lends itself well to an RTI framework, as this is engaging and motivating for students and frees up teacher time to work with students who may be experiencing difficulties. Because Accelerated Math automatically scores students’ assignments, the teacher can immediately and effectively act upon data from the program to ensure students are making progress toward goals.

Enhancements to Accelerated Math allow teachers to make true differentiation—the key to RTI—a reality, by processing performance data to support differentiated assignments, by helping the teacher create those assignments, and by automatically generating a flow of data to the teacher, student, and parent(s) that explains at a glance whether individual goals are being met.

Accelerated Math™ accolades
Accelerated Math is a standardized, reliable, valid, efficient, and cost-effective continuous progress-monitoring system. The program is one of several Renaissance tools that experts have deemed meet high scientific standards of quality, making it a perfect fit for screening and progress monitoring.

Since 2012, Accelerated Math has been highly rated for progress monitoring by the National Center on Intensive Intervention (NCII, 2015). NCII’s approach to intensive intervention is data-based individualization, which is a research-based process for individualizing and intensifying interventions through the systematic use of assessment data, validated interventions, and research-based adaptation strategies. In 2010, the National Dropout Prevention Center/Network determined Accelerated math has “strong evidence of effectiveness” for prevention and intervention at all grade levels.

In 2009, Accelerated Math was the first progress-monitoring tool to be highly rated as a mastery measure by the National Center on Response to Intervention (NCRTI). NCRTI defines progress monitoring as “repeated measurement of academic performance to inform instruction of individual students in general and special education. It is conducted at least monthly to: (a) estimate rates of improvement, (b) identify students who are not demonstrating adequate progress, and/or (c) compare the efficacy of different forms of instruction to design more effective individualized instruction.” In 2007, the National Center on Student Progress Monitoring—the predecessor to the NCRTI—determined that Accelerated Math met all criteria for scientifically based progress-monitoring tools.
Efficacy: Key research support for Accelerated Math™

When used with integrity, Accelerated Math has been shown to accelerate math achievement for all students. To ensure that teachers make the most of their Accelerated Math data and that students benefit to the greatest extent possible, fidelity of implementation is guided by research-based professional development promoting best practices in mathematics.

The large evidence base supporting Accelerated Math numbers 101 studies and reviews, including 33 experimental or quasi-experimental research studies—generally considered the strongest study designs—90 independent studies, and 22 articles published in peer-reviewed journals, and thus upheld to the highest scrutiny. A selection of this research is highlighted below in addition to a recent correlational study conducted by Renaissance.

**Trends in student outcome measures**

In a 2015 analysis using data from the 2013–2014 school year, Renaissance examined patterns of growth and expected college and career readiness according to the extent of individualized math practice accomplished by students. The study analyzed data for grades 1–12 from over 2.7 million students and 12,000 schools nationwide to compare independent math practice, as tracked by Accelerated Math, with the typical performance of students who do not use the program. Whether examined by grade or by populations of interest (students struggling with math, English learners, and students in free or reduced lunch programs), Accelerated Math was associated with better student performance and higher levels of annual growth (see figure 4). And notably, the better the program was used, the better the outcomes were for students.

**Figure 4. The better Accelerated Math is used, the more students grow and meet key benchmarks**

Note: Program use was voluntary (students were not recruited nor randomly assigned to a particular comparison group) and results should be considered correlational, not causal. While trends presented are helpful to understand patterns of growth at a high level, educators should rely most heavily on causal evidence, which generally requires an experimental or quasi-experimental design. The balance of this section presents such evidence for Accelerated Math.

**Accelerated Math impacts math performance of at-risk students**

In a study by Lambert, Algozzine, and McGee (2014), teachers in grades 2–5 at three Midwestern elementary schools implemented Accelerated Math as a progress monitoring and mastery measure tool with students at risk for school failure in 36 classrooms randomly assigned to either the treatment or control group. Students were tested with STAR Math at the beginning of the study for placement into Accelerated Math, then mid-year and end of year to measure growth. STAR Math results showed that the growth rate of the treatment group was significantly higher than the control group (see figure 5). In order to further evaluate the program’s effectiveness, treatment classes were categorized into either high- or low-implementation groups based on the level of implementation achieved. The researchers found that growth for the high-implementation group was significantly higher than for the low-implementation group.
Students, teachers, and schools as sources of variability, integrity, and sustainability in progress monitoring
To examine issues of variability, implementation integrity, and sustainability when implementing a progress-monitoring program, an investigation by Bolt, Ysseldyke, and Patterson (2010) followed some of the same schools, teachers, and students as studied by Ysseldyke and Bolt (2007; see study description on next page) for a second year as they began schoolwide implementation of Accelerated Math. The researchers also investigated the relationship between Accelerated Math implementation and positive student outcomes. The new study found significant variability in intensity of implementation across all three levels studied (student, teacher, and school); however, individual teacher implementation remained stable across the two years. Students of teachers who implemented Accelerated Math with greater fidelity experienced higher math gains on standardized assessments than other students; likewise, the relationship between Accelerated Math use and achievement was significant at the student level.

More students using Accelerated Math score Proficient or higher in math
While studying 360 elementary schools in Florida, Minnesota, New York, and Texas, researchers Burns, Klingbeil, and Ysseldyke (2010) found more students in schools using Accelerated Math scored in the Proficient or higher categories of their respective state mathematics tests. State reading test performance was controlled for as a covariate. The researchers also discovered a positive effect related to amount of time schools used the program. Schools working in Accelerated Math for 5 years or more outperformed those using it for less time or not at all. The results also showed no racial achievement gap among the treatment schools, whereas there was a gap in control schools.

Students who use Accelerated Math make gains regardless of instructional approach
Ysseldyke and Betts (2010) examined math curriculum data for nearly 2,000 classrooms assessing student achievement with STAR Math, some also using Accelerated Math software and some not. The researchers found that students who worked in Accelerated Math with the following curricula outperformed their peers who used only the curriculum: enVision Math, Everyday Mathematics, Holt McDougal, Macmillan/McGraw-Hill, and Saxon Math. The curricula examined represent both traditional and reform approaches to teaching math. It is noteworthy that Accelerated Math use resulted in significant positive effects with both approaches, suggesting that the program increases student achievement no matter the instructional approach that accompanies it.
Juniors at Arizona high school pass the AIMS after Accelerated Math intervention
Springer, Pugalee, and Algozzine (2007) conducted a randomized experiment with 28 at-risk high school juniors who did not pass the Arizona Instrument to Measure Standards (AIMS) test in grade 10. The control group participated in the school’s typical math classroom, while the experimental group used Accelerated Math. Figure 6 shows that afterwards, more students in the experimental condition (57%) were able to pass the state test than those in the control condition (14%). All students in the Accelerated Math classroom demonstrated positive gains on the test.

Figure 6. Accelerated Math students perform better on AIMS test than peers

Implementing Accelerated Math with fidelity is key to more growth
In a quasi-experimental study, Ysseldyke and Bolt (2007) examined the impact of Accelerated Math with nearly 2,000 elementary students from eight schools. Students from more than 100 classrooms spanning seven US states, including several in large cities, were randomly assigned to experimental (using Accelerated Math with existing curriculum) and control (using only existing curriculum) groups. The researchers discovered that students whose teachers used Accelerated Math as intended demonstrated greater gains on two standardized tests, TerraNova and STAR Math, than students with limited or no implementation (see figure 7).

Figure 7. High Accelerated Math implementation leads to greater TerraNova Gains

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).

Large-scale study reveals impact of Accelerated Math on grades 3–10 and subgroups
For this quasi-experimental study, Ysseldyke and Tardrew (2003, 2007) examined 2,200 students from 47 schools in 24 US states. They found that students using Accelerated Math in grades 3–10 achieved math gains from 7 to 18 percentile points higher than comparison students. In every grade and subgroup identified, such as eligibility for Title I and free or reduced lunch programs, students in Accelerated Math classes outperformed students not using the program (see figure 8). Students who closely followed Accelerated Math best practice recommendations by scoring higher than 85% correct and completing more subskills made even greater gains. Accelerated Math educators reported qualitative improvements too—teachers spent more time providing individual instruction, while students spent more time academically engaged, enjoyed math more, and took responsibility for their work. In total, 80% of Accelerated Math educators reported students were learning basic math skills better with Accelerated Math.
Schoolwide Accelerated Math implementation benefits Georgia students

In a quasi-experimental study, Holmes, Brown, and Algozzine (2006) examined the effectiveness of both Accelerated Math and Accelerated Reader with 2,287 students from four elementary schools (two rural, two urban) in central and northern Georgia. One school in each area was either a high or low implementer of Accelerated Math and Accelerated Reader. As shown in figure 9, results from the Criterion-Referenced Competency Tests indicated that students in two high-implementing schools outperformed students in two low-implementing comparison schools overall (effect size [ES]=0.65), and in math (ES=0.75), reading (ES=0.50), and language arts (ES=0.71). Teachers in all schools expressed positive attitudes toward both programs.

Figure 9. High Accelerated Math implementers outperform low on CRCT
Title I students see improvement in test scores after using Accelerated Math

The 870 students in this quasi-experimental study (Ysseldyke, Betts, Thill, & Hannigan, 2004) were a subset of students who participated in a large national experiment by Ysseldyke and Tardrew, 2003 (see p. 12). The students were in grades 3–6 from 47 schools in 24 US states. A two-group pretest/posttest comparison was used to evaluate whether students in a Title I program whose teachers used Accelerated Math would show greater gains in mathematics achievement than Title I students who received no intervention other than their regular math instruction. STAR Math results show that students using Accelerated Math significantly outperformed the comparison group, gaining an average 7.9 normal curve equivalents (NCEs) compared to the 0.3 NCEs earned by students not using Accelerated Math—a difference of 7.6 NCEs (see figure 10). Thus, evidence was found to support the claim that Accelerated Math can improve the math achievement of Title I students.

Figure 10. Title I students using Accelerated Math achieve more growth

Gifted and talented students score even higher after Accelerated Math practice

As with the previous study, the 843 students (in grades 3–6 from 47 schools) who participated in this quasi-experimental study (Ysseldyke, Tardrew, Betts, Thill, & Hannigan, 2004) were a subset of the sample from a large national experiment by Ysseldyke and Tardrew, 2003 (see p. 12). All students in Accelerated Math classrooms experienced greater gains in math achievement than those in comparison classrooms. In particular, of the 100 Gifted and Talented (GT) students in the sample, the GT students who used Accelerated Math advanced significantly more than those who did not. The STAR Math mean normal curve equivalent (NCE) gain for the experimental classrooms was 11.9 NCEs, a difference of 7.1 NCEs from the 4.8 gain in the control classrooms. As figure 11 shows, GT students using Accelerated Math also made bigger gains, obtained a higher percent correct on practice and test items, attempted more tests, and mastered more objectives than non-GT students using Accelerated Math.

Figure 11. GT students make greater gains with Accelerated Math

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).
Accelerated Math helps Minnesota students surpass district on NALT

This quasi-experimental study by Ysseldyke, Spicuzza, Kosciolek, and Boys (2003) examined the effects of Accelerated Math on math achievement and classroom behaviors known to be related to overall student achievement. Sixty-eight percent of students who participated in the study were eligible for free or reduced price lunch. The researchers assigned 160 students from three schools in a large, urban Midwestern district to classes to use Everyday Math (the district curriculum) with Accelerated Math or without. Students in Accelerated Math classrooms excelled in math achievement on STAR Math and the Northwest Achievement Levels Test (NAL T), including outperforming a district sample by 4.02 normal curve equivalents (NCEs) (see figure 12). In addition, ecological observation data indicated that Accelerated Math use resulted in increased time spent on classroom activities, which researchers identified as contributing to positive academic outcomes. The authors also found that Accelerated Math automates the application of evidence-based components of effective instruction.

Figure 12. Students using Accelerated Math outpace district average on NALT

![Figure 12](image)

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).

Accelerated Math practice allows Midwestern students to exceed national norms

In a quasi-experimental study, Ysseldyke, Spicuzza, Kosciolek, Teelucksingh, et al. (2003) examined the effect of Accelerated Math on overall student achievement at four schools in a large, urban Midwestern school district composed of approximately 75% minority students and 67% free or reduced price lunch eligibility. Researchers assigned 881 students in grades 3–5 to either use Accelerated Math with their regular curriculum, or just use the curriculum. Results showed that students at all ability levels who used the program demonstrated accelerated rates of performance compared to national norms, even after starting out below the norms before participating in the treatment classroom. As seen in figure 13, gains ranged from 3.4 to 10.8 NCEs on the Northwest Achievement Levels Test (NAL T), and increases were similar on the STAR Math assessment.

Figure 13. Students at all levels increase NALT scores with Accelerated Math

![Figure 13](image)

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).
Kansas high school students realize higher scores with Accelerated Math

A quasi-experimental study by Gaeddert (2001) involved 103 students at a Kansas high school who were enrolled in pre-algebra, algebra, and geometry classes. As shown in figure 14, results from the Stanford Achievement Test (SAT-9) showed the intervention (treatment) group gained, on average, 10.1 normal curve equivalents (NCEs), whereas the control group gained 3.5 NCEs. Students also responded to pre- and post-study attitudinal surveys, which showed improvement in attitudes toward math at the end of the study for students who used Accelerated Math and their parents.

**Figure 14. SAT-9 gains greater for Accelerated Math students**

<table>
<thead>
<tr>
<th>Subject</th>
<th>NCE Gain</th>
<th>Non-AM</th>
<th>AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-algebra</td>
<td>5.6</td>
<td>6.7</td>
<td>-6.1</td>
</tr>
<tr>
<td>Algebra</td>
<td>3.4</td>
<td></td>
<td>8.8</td>
</tr>
<tr>
<td>Geometry</td>
<td>16.6</td>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>Total</td>
<td>3.4</td>
<td>10.1</td>
<td></td>
</tr>
</tbody>
</table>

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).

Midwestern students at all achievement levels make greater gains with Accelerated Math

The majority of students participating in this quasi-experimental study at four urban Midwestern schools were eligible for free or reduced price lunch (Spicuzza et al., 2001). The 198 fourth and fifth graders studied were assigned to classrooms to use Accelerated Math or not. At all achievement levels, students in Accelerated Math classrooms demonstrated more growth on STAR Math and the Northwest Achievement Levels Test (NAL T) than students in non-Accelerated Math classrooms. The STAR Math adjusted mean for Accelerated Math students was 42.96 compared to 31.45 for comparison students. As seen in figure 15, on the NAL T, the adjusted normal curve equivalent (NCE) mean for Accelerated Math students was 51.3, and for the comparison group it was 46.6. The same students also outperformed a district sample of students in non-Accelerated Math classrooms.

**Figure 15. Accelerated Math use results in higher NAL T growth**

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).
Accelerated Math equals success for English learners in Minnesota

In this quasi-experimental study, Teelucksingh, Ysseldyke, Spicuzza, and Ginsburg-Block (2001) studied 201 English learners (ELs) in grades 4 and 5 from four Minneapolis schools to compare the math performance of students using Accelerated Math with best practices to a control group of students who did not receive the intervention. EL students in classrooms that used Accelerated Math in conjunction with their math curriculum gained 6.7 normal curve equivalents (NCEs) on the Northwest Achievement Levels Test (NALT) compared to EL students from the control group who gained 3.9 NCEs (see figure 16).

Figure 16. EL students make greater strides on NALT with Accelerated Math than peers

Note: Normal curve equivalent (NCE) scores are a way of representing percentile scores so they can be accurately averaged and compared with each other. Because NCEs are derived from percentiles, they measure growth in comparison to national norms. Positive NCE gains mean student achievement grew at a faster rate than the national average (an NCE gain of zero).

Accelerated Math promotes growth in grade 8 pre-algebra classrooms

Zumwalt (2001) conducted a 25-week study of 350 eighth-grade pre-algebra students in six schools in Idaho. In total, 94 students received traditional instruction, 162 were instructed using Accelerated Math, and 94 students used computer-aided instruction software from either Jostens Learning Corporation or Computer Curriculum Corporation (CCC). Figure 17 shows that on the Iowa Test of Basic Skills (ITBS) students who used Accelerated Math significantly outperformed students using Jostens, CCC, or traditional instruction. The significant differences between the Accelerated Math group (mean difference score: 29.26) and the Jostens-CCC (16.13) and traditional instruction (18.70) groups equated to almost a year of typical instructional growth and more than a year of growth, respectively. Lower performing students, in particular, benefited more from Accelerated Math.

Figure 17. Pretest to posttest change on ITBS is largest for students using Accelerated Math

Appendix A. Accelerated Math™ meets key components of effective instruction

The Accelerated Math program is based on evidence-based classroom strategies for ensuring that guided independent math practice accompanies direct instruction, supported by data and management features that accelerate math growth throughout the classroom and across grade and achievement levels.

In Table A1, the left column shows key evidence-based instructional practices based on Christenson and Ysseldyke (1989, 2002) as noted in many literature analyses (e.g., Christenson, Ysseldyke, & Thurlow, 1989; Gettinger & Stoiber, 1999) and infused with more recent research. The right column explains how these practices are incorporated into Accelerated Math.

Table A1. How Accelerated Math addresses research-based instructional practices

<table>
<thead>
<tr>
<th>Research-based instructional practice</th>
<th>How Accelerated Math aligns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional match and relevant practice</td>
<td>Initial placement in Accelerated Math is determined using results of a reliable and valid math assessment, such as STAR Math, and/or an Accelerated Math Diagnostic Test. Teachers then monitor progress of student work on Accelerated Math assignments, which helps to inform instruction and intervention.</td>
</tr>
<tr>
<td>Instructional expectations</td>
<td>Accelerated Math best practices recommend that teachers involve students in goal setting, including setting scientifically based goals for students using the STAR Math Goal-Setting Wizard. Immediate, informative feedback keeps students aware of their progress toward goals.</td>
</tr>
<tr>
<td>Classroom environment</td>
<td>The teacher is at the heart of the Accelerated Math classroom, delivering daily math instruction before students practice in the program. Accelerated Math excels at classroom management, by then completing administrative tasks and providing progress-monitoring information to help teachers provide timely feedback to students. Classroom routines include conferencing between students for additional instruction and intervention as needed as students use the program to practice.</td>
</tr>
<tr>
<td>Instructional presentation and student understanding</td>
<td>Accelerated Math supports best practices in mathematics classrooms such as clear communication of routines and expectations to students. The program facilitates teacher modeling and think alouds during teacher-to-student interaction, so students can hear and see the teacher’s thought processes while solving a problem. Likewise, student verbalization can be used to check their understanding of what the teacher demonstrated or provide insight into a student’s thinking.</td>
</tr>
<tr>
<td>Motivational strategies</td>
<td>Using Accelerated Math, teachers motivate students by providing opportunities for successful math practice. Accelerated Math keeps track of the number of subskills mastered, percent correct, completion of a library, and so forth, all of which can be used for goal setting to help motivate students to practice more.</td>
</tr>
<tr>
<td>Informed feedback</td>
<td>Immediate feedback is generated as students complete assignments, including corrective information about items answered incorrectly. Accelerated Math routines recommend that students rework missed problems on paper and to submit to the teacher for additional instruction or intervention as needed.</td>
</tr>
<tr>
<td>Academic engaged time</td>
<td>The Accelerated Math classroom is structured so that students use time productively, working on assignments or tests, submitting work to be scored, or conferring with the teacher for additional instruction and intervention as needed or with other students for peer-learning opportunities. Student routines and expectations are clearly communicated, so students always know where their efforts should be focused. If they need guidance, the teacher and classroom signage are available to provide direction.</td>
</tr>
<tr>
<td>Adaptive instruction</td>
<td>Through a specific practice, test, and review cycle, students are provided with multiple opportunities to practice and master an objective as well as ensure mastery over time through incremental review. Each subsequent assignment in Accelerated Math is based on a student’s prior work.</td>
</tr>
</tbody>
</table>
Appendix B. Fidelity of implementation

Effective instruction can only take place if tools used to support it are implemented with fidelity. To ensure that teachers make the most of the data from Accelerated Math and that students benefit to the greatest extent possible, implementation integrity is guided by research-based professional development promoting best practices in mathematics.

Research shows that high implementation of Accelerated Math promotes both personalized goal setting and appropriate, personalized practice (e.g., Nunnery & Ross, 2007; Ysseldyke & Bolt, 2007). Accelerated Math supports best practices, which include techniques for differentiating instruction, increasing and verifying academic engaged time (AET) for math skills practice, interpreting performance data, and monitoring the application of skills during student practice. The recommendations, which underscore the critical role the teacher plays in the effective use of Accelerated Math, are taught in a flexible series of in-person or web-delivered professional development sessions:

1. Math practice time—Teachers ensure students have an appropriate amount of time for guided independent mathematics practice. We recommend working toward a goal of 40 minutes of daily practice with Accelerated Math, which will put students on track to master, on average, four subskills per week.

2. Math success—Teachers ensure students are highly successful math learners, with an average percent correct of 75% or higher on practice assignments and 85% or higher on tests.

3. Appropriate math subskills—Teachers ensure students are practicing math subskills appropriate to their age and achievement level.

4. Progress monitoring—Teachers obtain information for progress monitoring from three sources: (1) daily, from direct teacher observation and conferences with students; (2) daily and weekly, from completion of student work in the software; and (3) periodically, 3–10 times per year, from a reliable and valid math assessment like STAR Math.

5. Personalized goals—Teachers motivate students by ensuring they are working toward personalized quality (average percent correct) and quantity (number of subskills mastered) goals.

6. Personalized instruction—Teachers use information from progress monitoring and goal setting to assess, inform, and tailor personalized instruction for each student.
References


